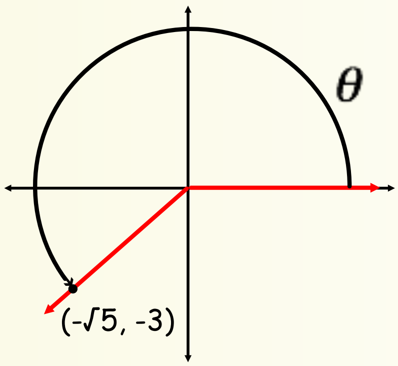
January 25, 2017

Precalc Warm Up # 6-3

Find the exact values of the 6 trig functions of θ



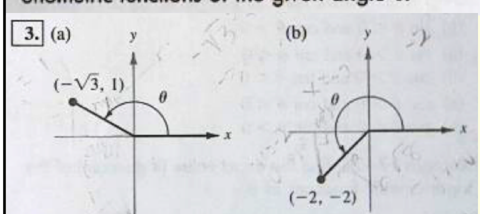
$\sin\theta$ $\csc\theta$

$\cos\theta$ $\sec\theta$

$\tan\theta$ $\cot\theta$

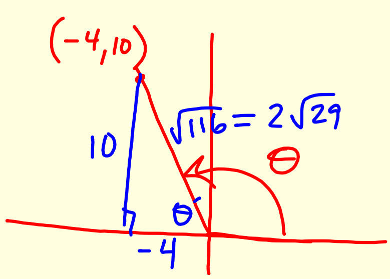
HW Questions: p. 339

In Exercises 1–4, determine the exact value of the six trigonometric functions of the given angle θ.



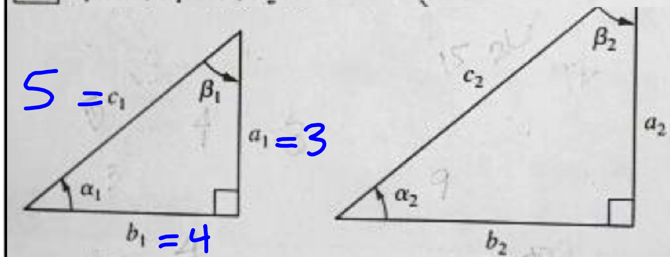
In Exercises 5–8, the given point is on the terminal side of an angle in standard position. Determine the exact value of the six trigonometric functions of the angle.

7. (a) $(-4, 10)$ (b) $(3, -5)$



In Exercises 9–12, use the two similar triangles in the accompanying figure to find (a) the unknown sides of the triangles and (b) the six trigonometric functions of the angles α_1 and α_2.

9. $a_1 = 3,\ b_1 = 4,\ a_2 = 9$

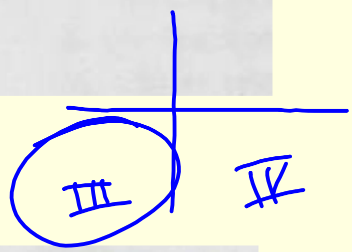


$$\frac{c_2}{5} = \frac{9}{3} \qquad \frac{b_2}{4} = \frac{9}{3}$$

In Exercises 13–16, determine the quadrant in which θ lies.

13. (a) $\sin\theta < 0$ and $\cos\theta < 0$
(b) $\sin\theta > 0$ and $\cos\theta < 0$

a)



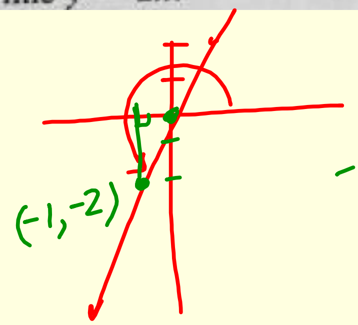
In Exercises 17–26, find the exact value (if possible) of the six trigonometric functions of θ.

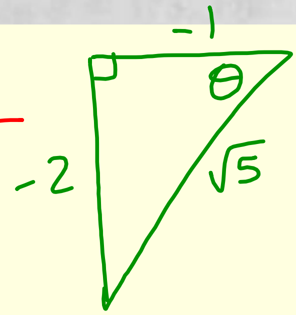
17. θ lies in Quadrant II, $\sin\theta = \frac{3}{5}$

21. $\sin\theta > 0$, $\sec\theta = -2 \longrightarrow \dfrac{2}{-1}$

$\dfrac{h}{a}$

25. The terminal side of θ is in Quadrant III and lies on the line $y = 2x$.

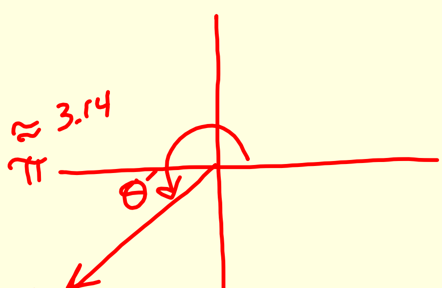




In Exercises 27–34, find the reference angle θ', and draw a sketch.

29. (a) $\theta = -245°$ (b) $\theta = -72°$

33. (a) $\theta = 3.5$ (b) $\theta = 5.8$



$\theta' = 3.5 - \pi$

≈ 0.358

In Exercises 35–44, evaluate the sine, cosine, and tangent of the given angles without using a calculator.

35. (a) $225°$ (b) $-225°$

39. (a) $\frac{4\pi}{3}$ (b) $\frac{2\pi}{3}$

In Exercises 45–52, use a calculator to evaluate the given trigonometric functions to four decimal places. (Be sure the calculator is set in the correct mode.)

45. (a) $\sin 10°$ (b) $\csc 10°$

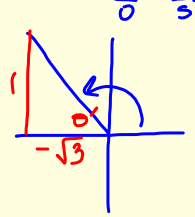
49. (a) $\cos(-110°)$ (b) $\cos 250°$

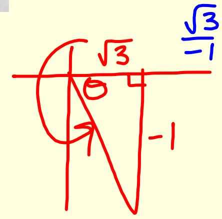
In Exercises 53–58, find two values of θ that satisfy the given equation. List your answers in degrees ($0° \le \theta < 360°$) and radians ($0 \le \theta < 2\pi$). Do not use a calculator.

53. (a) $\sin\theta = \frac{1}{2}$ (b) $\sin\theta = -\frac{1}{2}$

57. (a) $\tan\theta = 1$ (b) $\cot\theta = -\sqrt{3}$ $\frac{-\sqrt{3}}{1}$

$\frac{a}{o}$ $\frac{\cos\theta}{\sin\theta}$ $\frac{\sqrt{3}}{-1}$





In Exercises 65–68, evaluate the expression without using a calculator.

$\boxed{65.}$ $\sin^2 2 + \cos^2 2 = 1$

$\sin^2\theta + \cos^2\theta = 1$

$\boxed{69.}$ The average daily temperature (in degrees Fahrenheit) for a certain city is given by

$$T = 45 - 23\cos\left[\frac{2\pi}{365}(t - 32)\right]$$

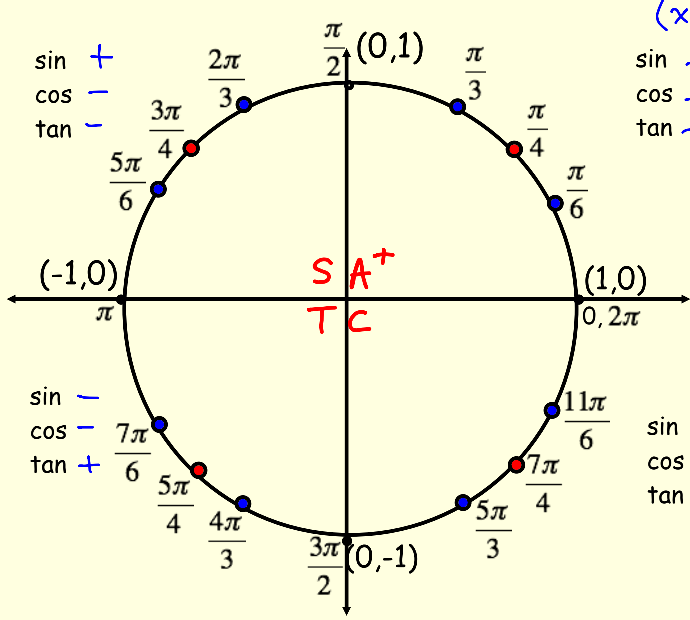
where t is the time in days, with $t = 1$ corresponding to January 1. Find the average temperature on (a) January 1, (b) July 4 ($t = 185$), and (c) October 18 ($t = 291$).

Positive or Negative?

$(\cos\theta, \sin\theta)$
(x, y)

sin +
cos −
tan −

sin +
cos +
tan +



sin −
cos −
tan +

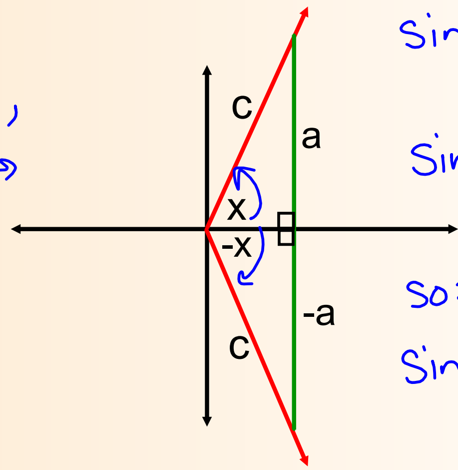
sin −
cos +
tan −

Is y = sin x odd or even?

Odd: $f(-x) = -f(x)$
(Origin symmetry)

Even: ~~$f(-x) = f(x)$~~
~~(y-axis symmetry)~~

Another way to think of it....
the inputs, x, are angles →



$\sin x = \frac{a}{c}$ (+)

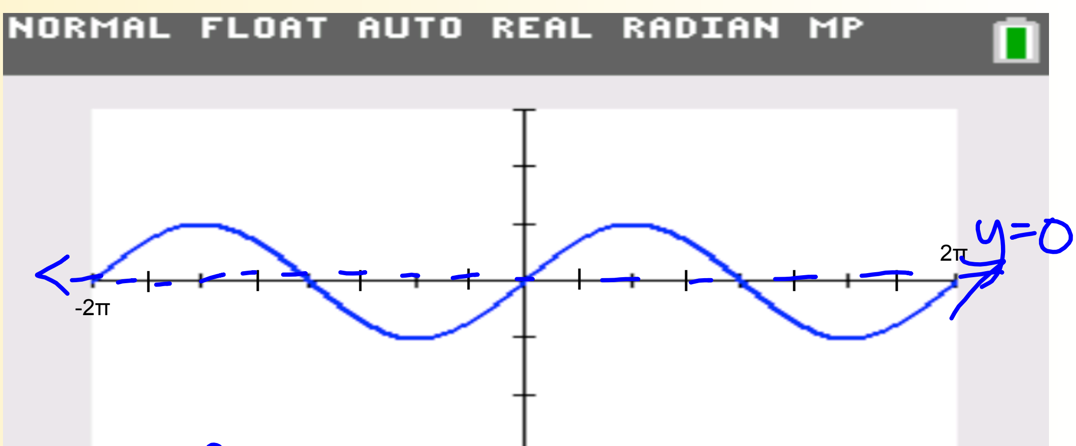
$\sin(-x) = \frac{-a}{c}$ (−)

so:
$\sin(-x) = -\sin x$

y = sin x is odd, so it has origin symmetry.

Graph y = sin x with grapher on [-2π, 2π].

Radian mode. x-scl? $\frac{\pi}{4}$



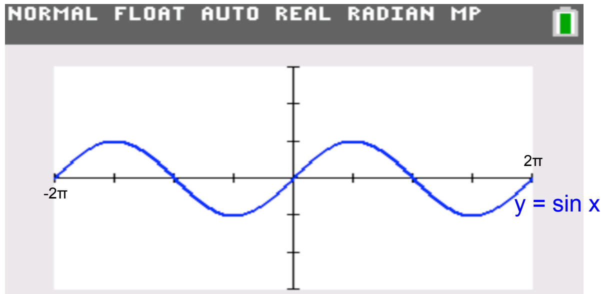
Period: 2π
(length of one cycle)

Line of oscillation:
(equation)

On grapher...

Explore y = a sin x

Try many values for a. Make sure to try negatives also.

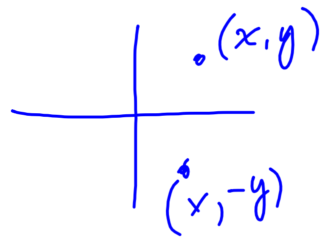


Summarize findings:

$a > 1 \longrightarrow$ vertical stretch

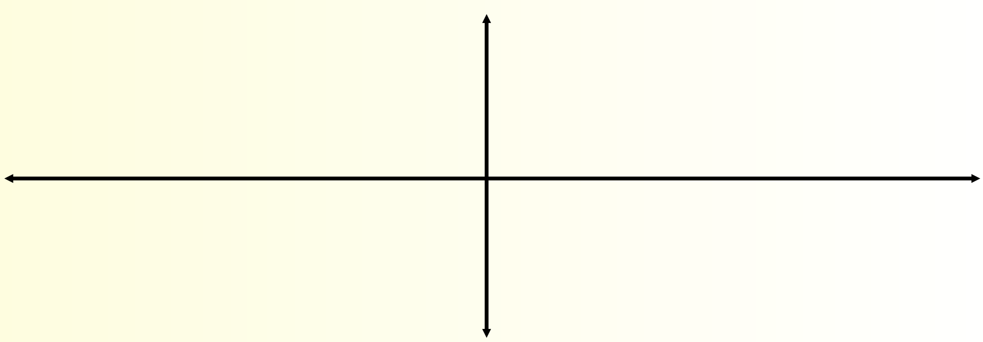
$0 < a < 1 \longrightarrow$ " compression

$a < 0 \longrightarrow r_x$



y = a sin x The a gives us a vertical stretch or compression

Without grapher, sketch 2 periods of y = 4 sin x

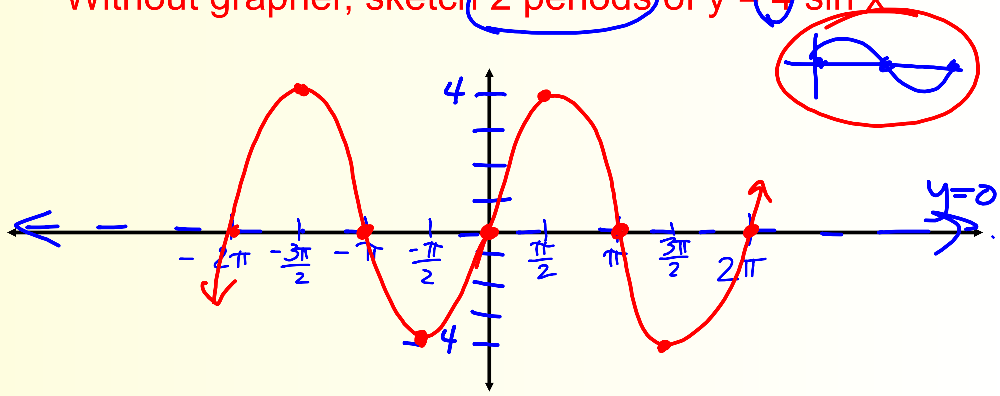


Amplitude = the distance from the line of oscillation.

A = |a| in y = a sin x

y = a sin x The a gives us a vertical stretch or compression Per 2π Amp=4

Without grapher, sketch 2 periods of y = 4 sin x



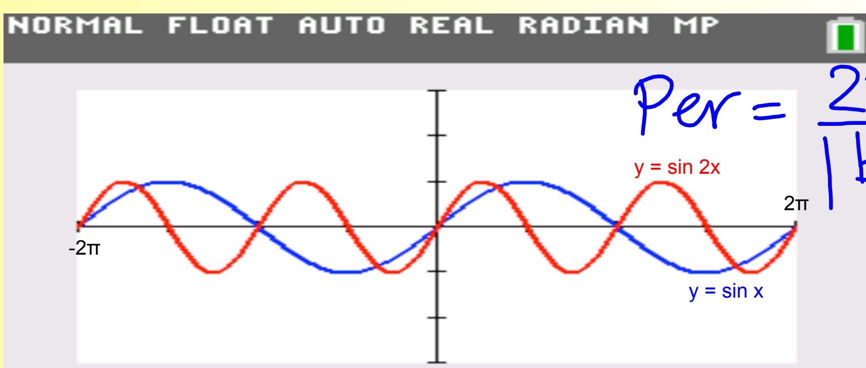
Amplitude = the distance from the line of oscillation.

$\boxed{A = |a|}$ in y = a sin x

Explore the effect of b on the graph of y = a sin bx

On grapher, graph both y = sin x and y = sin 2x

Try other values for b including negatives.



$\text{Per} = \frac{2\pi}{|b|}$

Summarize findings:

$b > 1$ horizontal compression

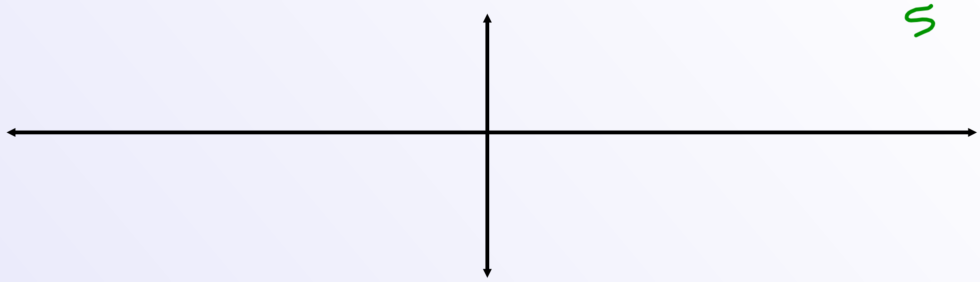
$0 < b < 1$ horizontal stretch

$b < 0$ r_y

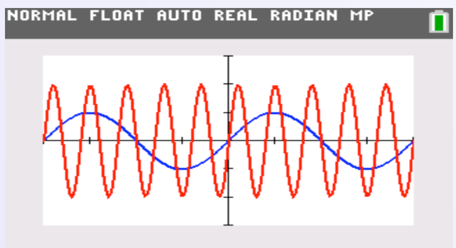
In y = a sin bx, b effects the ______

Without grapher, sketch 2 periods of y = 2 sin 5x

$\frac{2\pi}{5}$



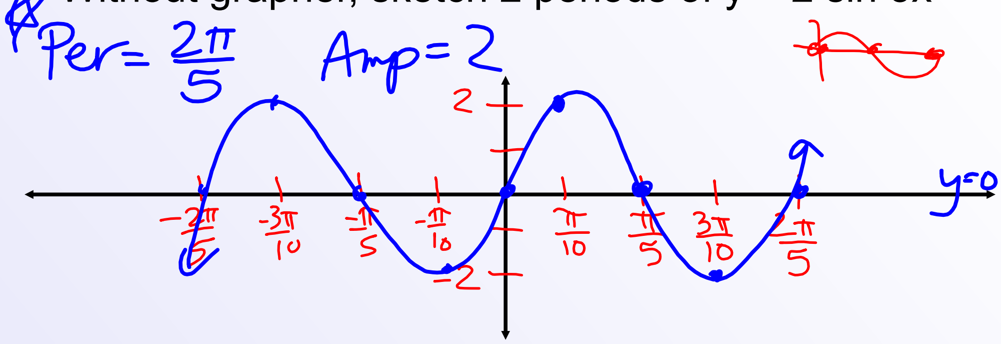
Then graph both y = sinx and y = 2 sin5x on grapher



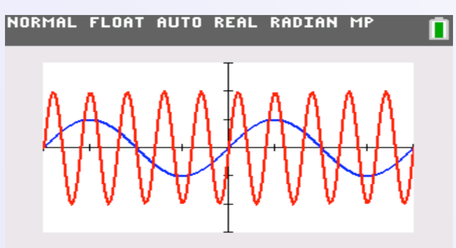
In y = a sin bx, b effects the Per $= \frac{2\pi}{|b|}$

Without grapher, sketch 2 periods of y = 2 sin 5x

*Per $= \frac{2\pi}{5}$ Amp = 2



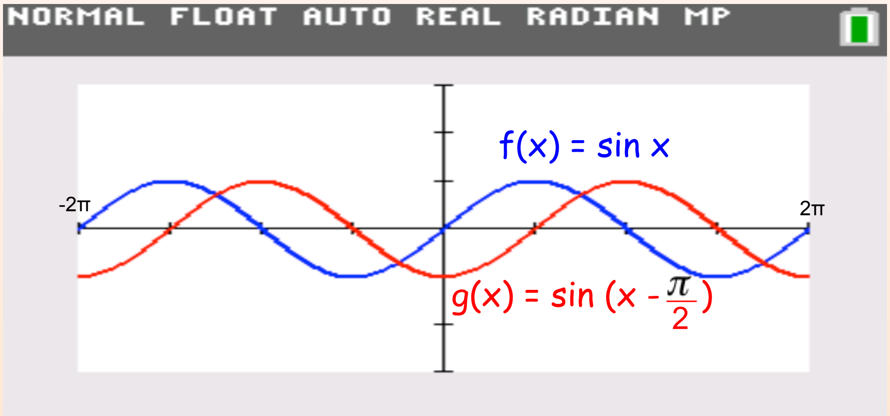
Then graph both y = sinx and y = 2 sin5x on grapher



What effect does c have? $y = a \sin b(x - c)$

$f(x) = \sin x$

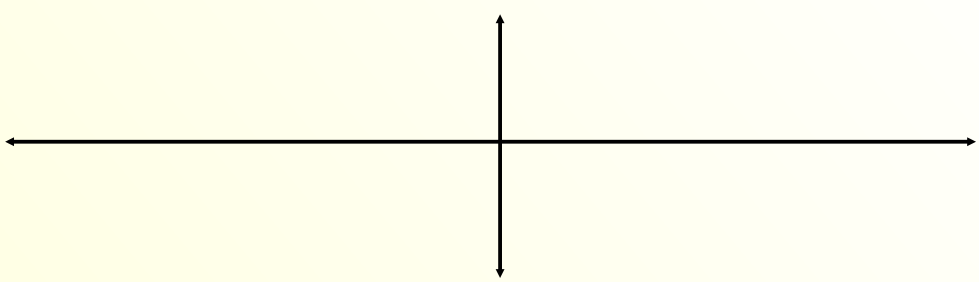
$g(x) = \sin (x - \frac{\pi}{2})$ How does g compare with f ?



Without grapher, sketch 2 periods of $y = \sin (2x + \frac{\pi}{2})$

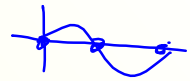
Period? Amplitude?

Transformations?



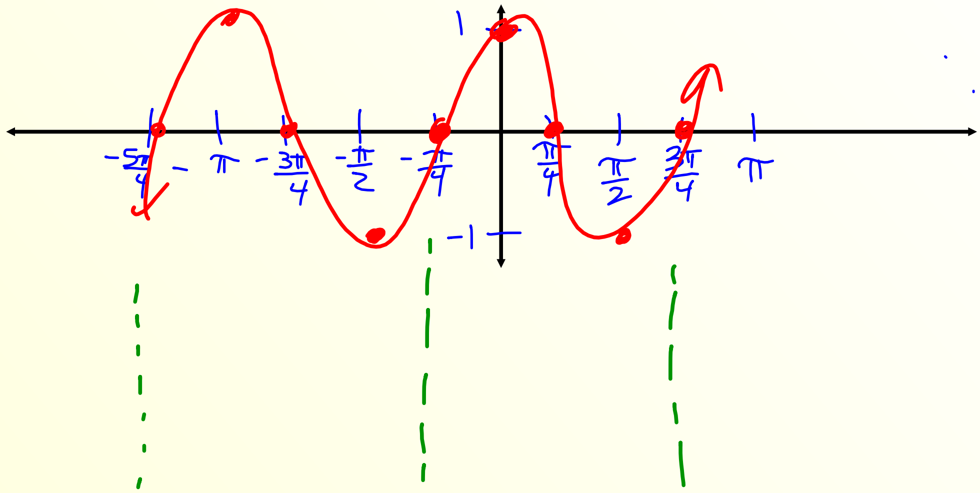
Without grapher, sketch 2 periods of $y = \sin\left(2x + \frac{\pi}{2}\right)$

Period? π Amplitude? 1 $y = \sin 2\left(x + \frac{\pi}{4}\right)$



Transformations?

Left $\frac{\pi}{4}$



Summary: $y = a \sin b(x - c) + d$

a effects:

b effects:

c effects:

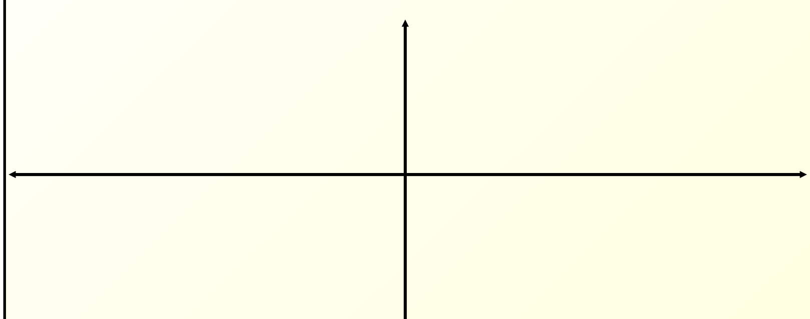
d effects:

When is there a reflection over x axis? y axis?

Without your grapher, sketch of $y = -4\sin(3x + \pi) + 2$

Per: Amp:

Transformations:



Summary: $y = a \sin b(x - c) + d$

a effects: Amp $= |a|$

b effects: Per $= \frac{2\pi}{|b|}$

c effects: horizontal translation

d effects: vertical translation

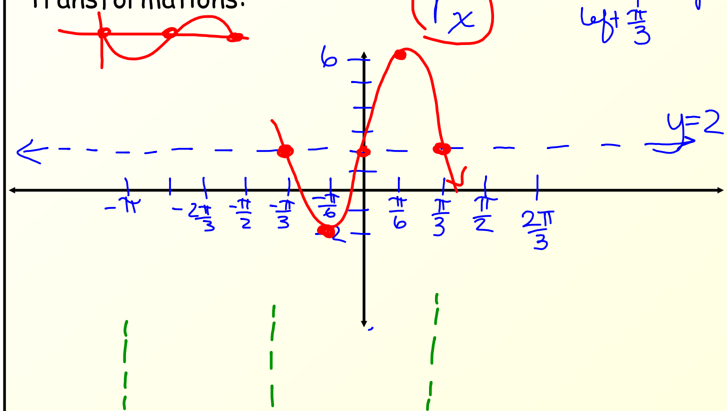
When is there a reflection over x axis? y axis?

when $a < 0$ $b < 0$

Without your grapher, sketch of $y = -4\sin(3x + \pi) + 2$

Per: $\frac{2\pi}{3}$ Amp: 4 $= -4 \sin 3(x + \frac{\pi}{3}) + 2$

Transformations: r_x left $\frac{\pi}{3}$ up 2



With grapher: $y = \sin(\frac{\pi}{2} - x)$

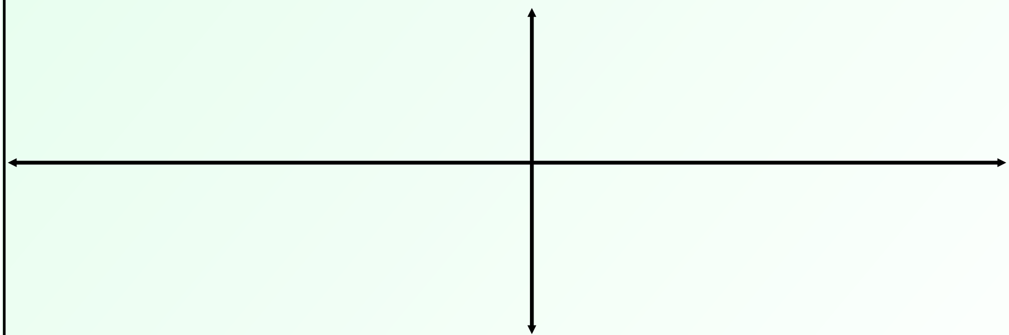
Per: Amp: Transformations:

What does $y = \sin(\frac{\pi}{2} - x)$ equal?

Sketch two periods of y = - 3 - cos($\frac{\pi}{2}$ - 2x)

Per: Amp:

Transformations:



Check with your grapher.

Sketch two periods of y = - 3 - cos($\frac{\pi}{2}$ - 2x)

Per: π Amp: 1

down 3

r_x

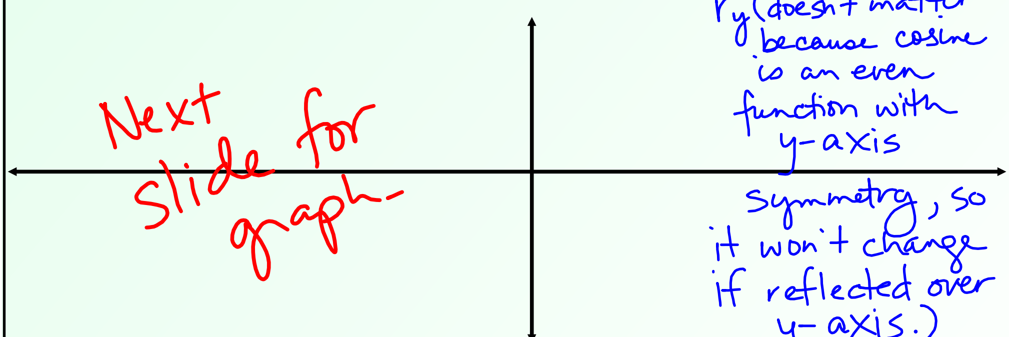
Rt $\frac{\pi}{4}$

$\cos\left[-2\left(x - \frac{\pi}{4}\right)\right]$

r_y (doesn't matter because cosine is an even function with y-axis symmetry, so it won't change if reflected over y-axis.)

Transformations:

Next slide for graph.

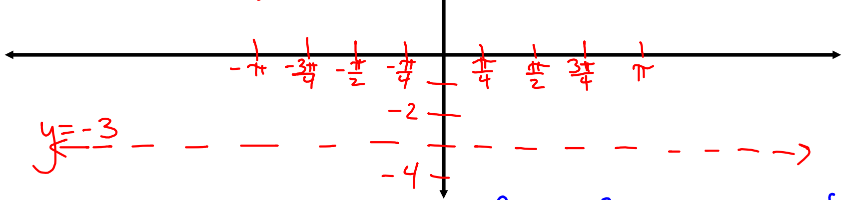


Check with your grapher.

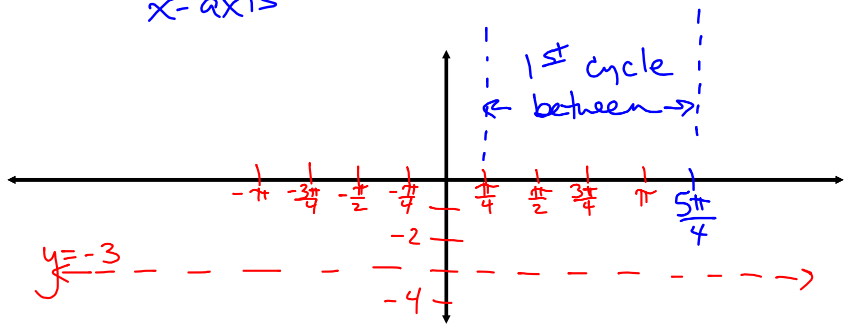
Step 1: Draw new line of oscillation @ $y=-3$

2: Mark off y-axis from there (Amp $=1$)

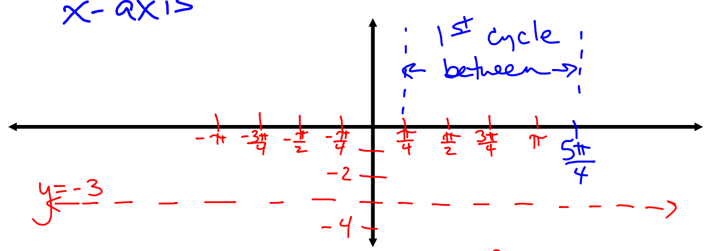
3: Mark off x-axis (Per $=\pi$)



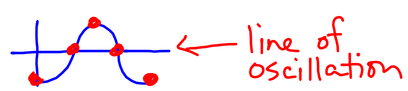
Step 4: consider horizontal shift: Rt $\frac{\pi}{4}$, adjust x-axis

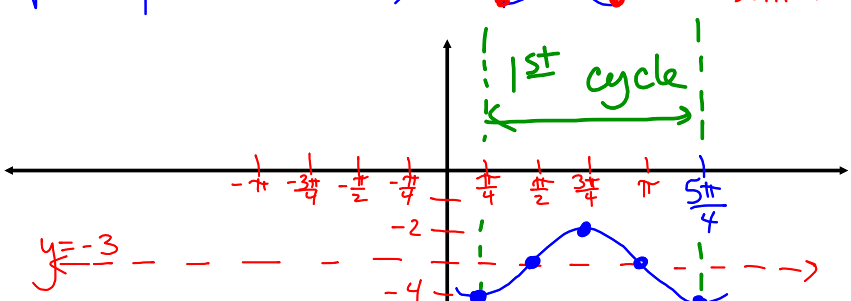


Step 4: consider horizontal shift: Rt $\frac{\pi}{4}$, adjust x-axis



Step 5: plot cosine r_x:





Last step: repeat pattern for a 2nd cycle. ☺

Quiz: SL 9.1, 9.7 and PC 5.1 - 5.4

Things to know for the quiz tomorrow:

Trig ratios; how to use them in a right triangle
Radian measures
Special triangles
Arc length and area of a sector
Using trig ratios for angles that are not acute

HW: PC book, p. 352 boxed

Sketch all graphs by hand, without your grapher! Label both axes and all 4 positions of at least one cycle.

Quiz tomorrow: SL 9.1, 9.7
PC 5.1 - 5.4